Opinion dynamics on directed small-world networks

Luo-Luo Jiang[1], Da-Yin Hua[2], Jun-Fang Zhu[1], Bing-Hong Wang[1],* and Tao Zhou[1,3]

[1] *Department of Modern Physics, University of Science and Technology of China, Hefei 230026, PR China*
[2] *Department of Physics, Ningbo University, Ningbo Zhejiang 315211, PR China*
[3] *Department of Physics, University of Fribourg, Chemin du Muse 3, CH-1700 Fribourg, Switzerland*
(Dated: February 2, 2008)

In this paper, we investigate the self-affirmation effect on formation of public opinion in a directed small-world social network. The system presents a non-equilibrium phase transition from a consensus state to a disordered state with coexistence of opinions. The dynamical behaviors are very sensitive to the density of long-range interactions and the strength of self-affirmation. When the long-range interactions are sparse and individual generally does not insist on his/her opinion, the system will display a continuous phase transition, in the opposite case with high self-affirmation strength and dense long-range interactions, the system does not display a phase transition. Between those two extreme cases, the system undergoes a discontinuous phase transition.

I. INTRODUCTION

Recently, much effort has been devoted to the studies of opinion dynamics [1, 2, 3]. The Sznajd model [4, 5], Galam's majority rule [6, 7], and the Axelrod multicultural model [8] describe opinion dynamics as individuals follow their nearest neighbors' opinion. However, the real-life system often seems a black box to us: the outcome can be observed, but the hidden mechanism may be not visible. If we see many individuals hold the same opinion, we say *The Spiral of Silence* phenomenon [9] occurs. It is common in real world that people adhere to their own opinion even opposite to most of their friends [10, 11, 12], which we call *self-affirmation* of individuals. Traditional opinion models fail to account for the self-affirmation effect of individuals. In our early work, the influence of inflexible units has been investigated in a simple social model [13]. It is found that this kind of effect can lead to a nontrivial phase diagram.

In the real world, interactions between individuals are not only short ranged, but also long ranged [14, 15]. The interaction usually takes on directed feature, which means an individual who receives influence from a provider may not effect the provider. We use directed links to represent this kind of relation between individuals in the directed small-world networks proposed by Sánchez *et al* [16]. In this paper, we present an opinion dynamics model including individual self-affirmation psychological feature and directed long-range correlation between individuals. The parameter space can be roughly divided into three regions, in which, respectively, we observed continuous phase transition, discontinuous phase transition and no phase transition.

II. MODEL

In this section, we introduce a directed small-world network model and an opinion dynamics model. We start with a two-dimensional regular lattice, in which every node is connected with adjacent four nodes inwardly and outwardly respectively, then, with probability p, rewire each link connected outwardly with a neighbor node to a randomly chosen nonadjacent node. In this way, as shown in Fig. 1, a directed network with a density p of long-range links is obtained. In the network, nodes represent individuals in the social system and the outward links represent the influence from others. Each node conects with four nodes outwardly which are called mates of the node. Long-range links are used to describe the long-range correlations between individuals.

In the network, state of each node represents the viewpoint of the corresponding individual, which evolves according to social process, determined not only by other correlative surrounding effects but also by its own character. To illustrate these cases simply, it is supposed that there are two kinds of possible opinions in the system, just as the agreement and disagreement in the election, and each individual takes only one of them. Therefore, the state of a node i can be described as σ_i, $\sigma_i \in \{+1, -1\}$. We describe the difference of σ_i between its mates' state by the function $W(\sigma_i) = 2\,\sigma_i \sum_{j=1}^{4} \sigma_j$, in which σ_j $(j = 1, 2, 3, 4)$ are states of mates of node i. In addition, q $(0 < q \le 1)$ is used to describe the probability, with which individuals follow their mates' dominant opinion. Meanwhile $1 - q$ represents the self-affirmation probability of individuals, with which an individual insists on his/her own opinion though it is opposite to most of his/her mates.

According to the illumination above, we introduce the dynamic rule as follows: $W(\sigma_i) > 0$ indicates that σ_i is the same as the majority of σ_j $(j = 1, 2, 3, 4)$ and σ_i overturns with probability $\exp[-W(\sigma_i)/T]$ which depends on a temperature-like parameter T. $W(\sigma_i) < 0$ represents that σ_i is opposite to the majority of σ_j $(j = 1, 2, 3, 4)$,
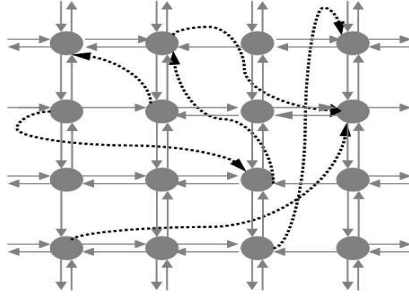
*Electronic address: bhwang@ustc.edu.cn

FIG. 1: Illustration of the structure of a directed small-world network for $p = 0.1$ [16].

and σ_i overturns with probability q. When $W(\sigma_i) = 0$, we consider that the state of node i overturns with probability q also. So that the overturning probability $P(\sigma_i)$ of σ_i is given by

$$P(\sigma_i) = \begin{cases} \exp[-W(\sigma_i)/T] & \text{, for } W(\sigma_i) > 0 \\ q & \text{, for } W(\sigma_i) \leq 0. \end{cases} \quad (1)$$

From the dynamical rule (1), we can see that, when $q = 1$, the current model restores to the network-based Ising model [16]. However, our model is non-equilibrium because the overturning probability of a state does not satisfy the detailed equilibrium condition.

III. SIMULATIONS

In order to describe the evolution process of the model, we employ a magnetization like order parameter

$$m = | \frac{1}{L^2} \sum_{i=1}^{L^2} \sigma_i |, \sigma_i \in \{+1, -1\}. \quad (2)$$

The network size is $L \times L$ and m represents the absolute average value of the states of all nodes. An extensive Monte Carlo numerical simulation has been performed on our model with a random initial configuration and a periodic boundary. Results are calculated after the system reaches a non-equilibrium stationary state. In order to reduce the occasional errors, for network size $L = 16$, 32, 64, and 100, we have averaged the result over 40000, 10000, 2000, and 1000 runs, respectively, with different network structures under different random initial configuration. Obviously, when $\langle m \rangle$ tends to 1, the system enters into an ordered state, i.e. individuals in the system reach the opinion consensus. Meanwhile, if the system stays in a disordered state, the order parameter scales as $\langle m \rangle \sim \frac{1}{L}$. As shown in Fig. 2, the system reaches an ordered state when T is less than a critical temperature T_c. The system displays a continuous phase transition for $p = 0.1$ and $q = 0.9$, while a discontinuous phase transition for $p = 0.9$ and $q = 0.9$. From the probability density functions (PDFs) of the order parameter near the

phase transition point of the phase diagram (p, T), one can distinguish between the continuous phase transition and discontinuous phase transition clearly. According to PDFs inserted in the upper and lower of Fig. 2, It is found that the most probable values of m, which correspond to the highest peaks of PDFs, jump little from nonzero to zero in continuous phase transitions from Fig. 2a to Fig. 2b, while largely in discontinuous ones from Fig. 2c to Fig. 2d. It seems that the long-range correlations can change the nature of phase transition.

Evidently, the system varies from the continuous phase transition to discontinuous phase transition when the density of directed long-range connections is high enough for $q = 0.9$. It is natural to ask how these topology structures influence the opinion dynamics. To solve this problem, we define the domain size s as the number of neighborhood nodes in the same state. As showed in Fig. 3a, it is found that the domain size s distributes in a power law, $g(s) \sim s^{-\tau}$ for $s \ll L^2$ at the critical point, where $g(s)$ is the probability function. One can find that there is a local maximum probability of large domain for $p = 0.1$ compared to the size distribution for $p = 0.9$ in Fig. 3a. Smaller p indicates more localized interactions between individuals, and a large domain emerges more easily. Besides, we calculate the number of time steps, t, during which an individual holds the same opinion. As shown in Fig. 3b, one can find that individuals change their own opinion for $p = 0.9$ more frequently than for $p = 0.1$ at $T = 0.1$, and the probability of t obeys a power-law distribution $f(t) \sim t^{-\gamma}$ $(t < t_0)$ for $p = 0.1$. Clearly, p plays the key role in determining the communication strength between different opinion domains, and individuals change their own opinions more frequently because of the effect of long-range connections between different opinion domains.

Fig. 4a, 4b, and 4c show the phase diagram of opinion dynamics determined by the network structure parameter p as well as the individual self-affirmation psychology characteristic parameter $1 - q$. In Fig. 4a for q=0.9, the system displays continuous phase transition for $p < p_c$, while discontinuous phase transition for $p \geq p_c$. The system displays discontinuous phase transition for $q = 0.5$ in Fig. 4b. The system displays discontinuous phase transition for $p < p_0$ and $q = 0.3$ in Fig. 4c, while the system does not have a phase transition for $p > p_0$ and $q = 0.3$ in
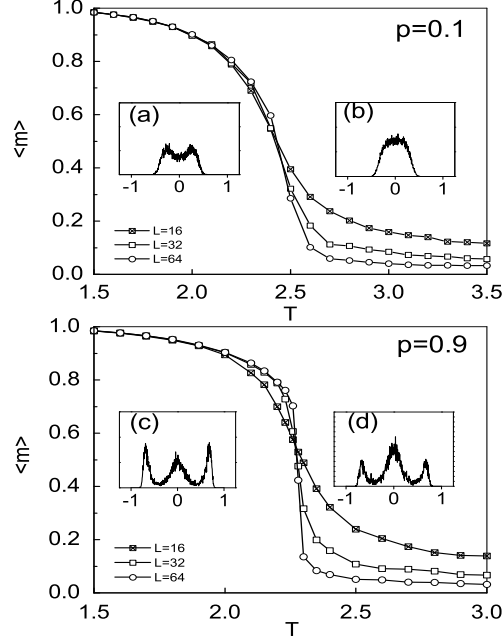
FIG. 2: $\langle m \rangle$ varies with T for different system sizes. The upper and lower plots are for $p = 0.1$ and $p = 0.9$, with $q = 0.9$ fixed. Insets are PDFs nearby the phase transition point: (a) $T \to T_c^-$, (b) $T \to T_c^+$, (c) $T \to T_c^-$, (d) $T \to T_c^+$.
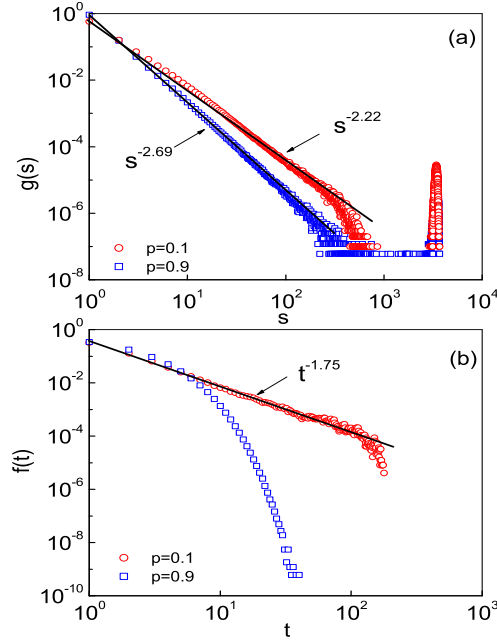

FIG. 3: (Color online.) The distribution of domain size $g(s)$ (a) and opinion holding time $f(t)$ (b) in different networks with $q = 0.9$ fixed, (a) for $T = T_c$ and (b) for $T = 0.1$. The data points are obtained from 10^5 samples with fixed network size, $L = 64$.

Fig. 4c. As shown in Fig. 4d, the continuous phase transition takes place in the area I, the discontinuous phase transition appears in the area II and the system stays in disordered state without phase transition in the area III. When both the parameters p and $1 - q$ are large enough, indicating weak interaction between individuals in both local and global levels, the system keeps in disordered completely at any temperature, i.e. the phase transition can not take place in the system, as in the area III of Fig. 4d.

A finite-size scaling analysis is employed to study the critical behavior of continuous phase transition for $p = 0.1$ and $q = 0.9$. In the neighborhood of the critical point T_c, $\langle m \rangle \propto (T_c - T)^{\beta}$, $(T < T_c)$, where β is the order parameter exponent. Besides, when T is near to critical point T_c of the second order phase transition, a character length scale ξ denotes the correlation length in space. $\xi \propto (T_c - T)^{-\nu}$, $(T < T_c)$, where ν is a correlation length exponent in the space direction. At critical point, various ensemble-averaged quantities depend on the ratio of system size and the correlation length L/ξ. Therefore, the order parameter $\langle m \rangle$ satisfies the scaling law in the neighborhood of the critical point: $\langle m \rangle \propto L^{-\beta/\nu} f[(T_c - T)L^{1/\nu}]$. At T_c, $\langle m \rangle \propto L^{-\beta/\nu}$, and we obtain $\beta/\nu = 0.530(5)$ for $p = 0.1$ and $q = 0.9$ in Fig. 5(a). Fig. 5(b) reports $\langle m \rangle L^{\beta/\nu}$ versus $(1 - T/T_c)L^{1/\nu}$ on a double-logarithmic plot for $q = 0.1$ and q=0.9. It is shown that with the choices $\beta/\nu = 0.530(5)$ and $\nu = 0.92(1)$ the data for different network sizes are well collapsed on a single master curve [17]. The slope of the line is $\beta = 0.488 \pm 0.005$, which gives the asymptotic behavior for $\langle m \rangle L^{\beta/\nu}$ as $L \to \infty$. So that, we have $\beta = 0.488(5)$, $\nu = 0.92(1)$.

IV. CONCLUSION

In conclusion, the effect of directed long-range links between individuals on the opinion formation is system-

atically explored. The results show that the system takes on a non-equilibrium phase transition from a consensus state to a state of coexistence of different opinions. With increasing density of long-range links, a continuous phase transition changes into a discontinuous one. The reason why the phase transition behavior varies is that the long-range links make individuals change their own opinion more frequently. It is worth mentioning that the system keeps in a disordered state when there are sufficient long-range links. Those long-range interactions break the possibly local order, thus hinder the global consensus.

In opinion dynamics, the self-affirmation psychology character sometime may lead to polarized decision [18, 19]. Moreover, interaction between individuals in social system depends on the topology of social networks [20, 21]. In macroscopic level, the opinion dynamics is highly affected by social structure, while in the microscopic, it is sensitive to the dynamical mechanism of individual. Our work shows a systematic picture of opinion dynamics, and provides a deep insight into effects of these two factors.

[1] Q. Michard and J.-P. Bouchaud, Eur. Phys. J. B **47**, 151 (2005).

[2] D. Stauffer and M. Sahimi, Eur. Phys. J. B **57**, 147 (2007).

[3] M. Gabbay, Physica A **378**, 118 (2007).

[4] K. Sznajd-Weron, and J. Sznajd, Int. J. Mod. Phys. C **11**, 1157 (2000).

[5] D. Stauffer, and P. M. C. de Oliveira, Eur. Phys J. B **30**, 587 (2002).

[6] S. Galam, Physica A **285**, 66 (2000).

[7] P. L. Krapivsky, and S. Redner, Phys. Rev. Lett. **90**, 238701 (2003).

[8] R. Axelrod, J. Conflict Res. **41**, 203 (1997).

[9] E. Noelle-Neumann, J. Commun. **24**, 43 (1974).

[10] T. Zhou, P.-L. Zhou, B.-H. Wang, Z.-N. Tang, and J. Liu, Int. J. Mod. Phys. B **18**, 2697 (2004).

[11] T. Zhou, B.-H. Wang, P.-L. Zhou, C.-X. Yang, and J. Liu, Phys. Rev. E **72**, 046139 (2005).

[12] J. Ma, P.-L. Zhou, T. Zhou, W.-J. Bai, and S.-M. Cai, Physica A **375**, 709 (2007).

[13] L.-L. Jiang, D.-Y. Hua, and T. Chen, J. Phys. A: Math. Gen. **40**, 11271 (2007).

[14] D. J. Watts and S. H. Strogatz, Nature **393**, 440 (1998).

[15] R. Albert and A.-L. Barabási, Rev. Mod. Phys. **74**, 47 (2002); R. Pastor-Satorras, and A. Vespignani, *Evolution and structure of the Internet* (Cambridge University Press, Cambridge, 2004).

[16] A. D. Sánchez, J. M. López, and A. Rodríguez, Phys. Rev. Lett. **88**, 048701 (2002).

[17] D.-Y. Hua, S.-J. Shao, and S. Lin, Phys. Rev. E **69**, 046114 (2004).

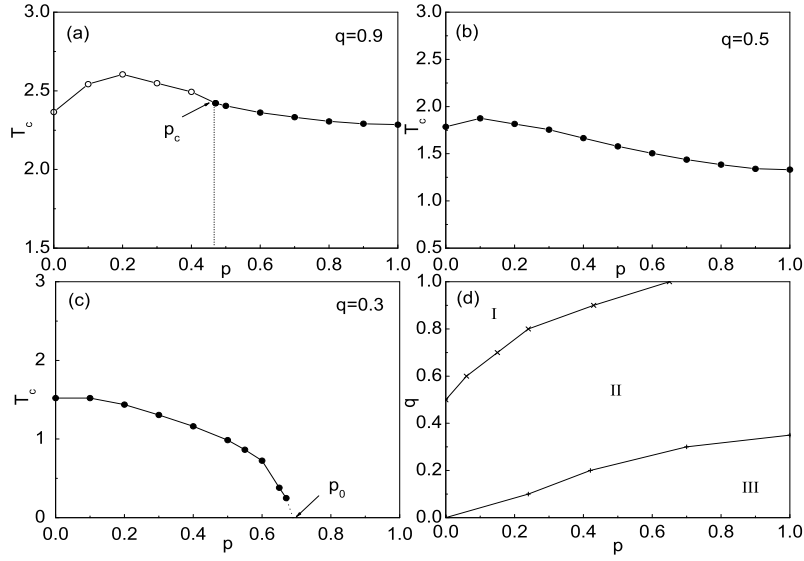[18] M. Blattner, Y.-C. Zhang, and S. Maslov, Physica A **373**,

FIG. 4: The phase diagram of the model in the $p - q$ plane. Points are numerical determinations of the critical temperatures T_c for different degrees of topological disorder p. The transition is continuous for open circles points, while discontinuous for filled circles points. Plot reports the full picture of phase diagram: the system displays continuous phase transition in region I, discontinuous phase transition in region II, and no phase transition in region III.
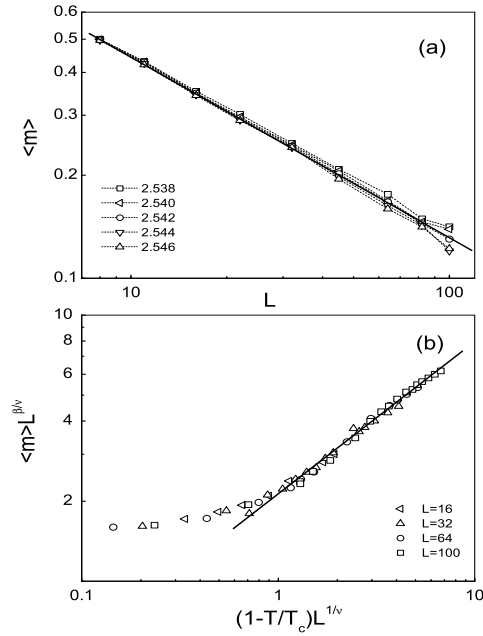

FIG. 5: Finite size scaling of continuous phase transition for $p = 0.1$ and $q = 0.9$. (a) A log-log plot of the order parameter $\langle m \rangle$ against L. (b) Double logarithmic plot of $\langle m \rangle L^{\beta/\nu}$ versus $(1 - T/T_c)L^{1/\nu}$ for $L = 16, 32, 64,$ and 100.

753 (2007).

[19] Y.-C. Zhang, M. Medo, J. Ren, T. Zhou, T. Li, and F. Yang, EPL. **80**, 68003 (2007).

[20] A. O. Sousa, and J. R. Sánchez, Physica A **361**, 319 (2006).

[21] G. Weisbuch, Eur. Phys. J. B **38**, 339 (2004).